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Heat transfer over a continuously moving plate embedded in non-Darcian porous medium

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Abstract

Heat transfer over a continuously moving plate embedded in non-Darcian porous medium is considered. Boundary layer equations are derived. The resulting approximate nonlinear ordinary differential equations are solved numerically. Graphical results for the velocity, temperature, Nusselt number are presented and discussed. The results of the present study show that the exponential parameters in the velocity and heat flux functions affected the heat transfer coefficient. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

A continuous moving surface through a porous medium has many applications such as geothermal reservoirs, and petroleum industries (petroleum drilings).

The simultaneous effects of fluid inertia force and boundary viscous resistance upon flow and heat transfer in a constant porosity porous medium were analyzed by Vafai and Tien [1]. Non-Darcian inertia effects on heat transfer were considered by many investigators [2–4].

In most of the previous studies, the plate was assumed to be stationary and the fluid moved over this plate. In other studies, the plate was moved in stationary fluid.

The similarity solutions for the governing ordinary differential equations of the boundary layer corresponding to a stretching surface have been studied by Ali [5]. Recently, Elbashbeshy [6] studied the heat transfer problem for a continuous stretching surface with variable heat flux. The heat transfer in boundary layer on an exponentially stretching surface are examined both analytically and numerically by Magyari and Keller [7], see Fig. 1.

In our knowledge, there has not been any reported study concerning heat transfer over a continuously moving plate embedded in non-Darcian porous medium.

The present work is to study the problem discussed by Elbashbeshy [6] to include a uniform porous medium (non-Darcian).

2. Formulation of the problem

We consider a steady, two-dimensional flow of a fluid past a continuously moving plate with variable surface heat flux $q_w(x) = Ax^{\lambda}$ and velocity $u_w = u_0 x^M$ (where A, λ, u_0 and M are constants) immersed in a fluid-saturated porous medium. The origin is located at the spot through which the horizontal plate is drawn in the fluid medium. The x-axis is chosen along

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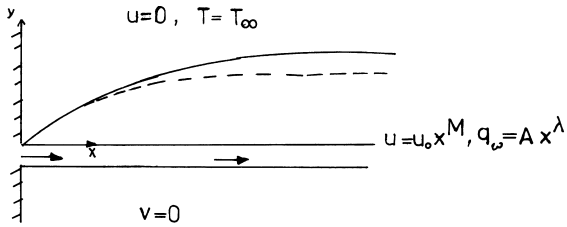

Fig. 1. Schematic of flow induced by a stretched surface.

the horizontal plate and y-axis is taken normal to it. We assume that: (1) The convective fluid and porous medium are in local thermal equilibrium. (2) Variable porosity and thermal dispersion effects are neglected. Upon treating the fluid saturated porous medium as a continuum (see Ref. [1]), including the non-Darcian inertia effects, the boundary layer form of the governing equations can be written as (see Refs. [1] and [6])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\rho}{\varepsilon^2}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\mu}{\varepsilon}\frac{\partial^2 u}{\partial y^2}-\frac{\mu_e}{K}u-Cu^2 \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_e\frac{\partial^2 T}{\partial y^2} \tag{3}$$

and associated boundary conditions

$$y=0:\quad u=u_0x^M,\quad v=0,\quad -k\frac{\partial T}{\partial y}=q_w(x)=Ax^{\lambda}$$

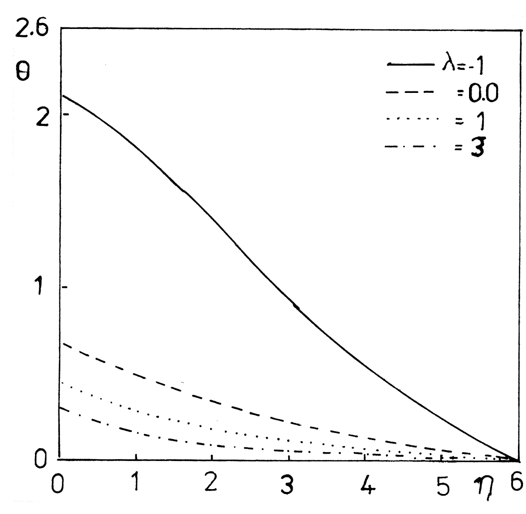

Fig. 2. Temperature profiles as a function of η for various values of λ at $\varepsilon=0.45$, $\gamma=2.01$, $M=0.5$, $\zeta=3$ and $Pr=3$.

Table 1
$Nu/\sqrt{Re}$ for variable surface heat flux for various values of λ at $M=0.5$, $Pr=3$, $\gamma=2.01$ and $\zeta=3$

$Nu/\sqrt{Re}$	0.4102	1.2896	1.9712	3.0174
λ	−1.0	0.0	1.0	3.0

$$y\to\infty:\quad u=0,\quad T=T_\infty \tag{4}$$

where u and v are the velocity components, x and y directions; ε and K are the porosity and permeability of porous medium, respectively; C is the transport property related to the inertia effect, μ is the viscosity, T is the temperature, α_e is the effective thermal diffusivity of the saturated porous medium, k is the thermal conductivity, T_∞ is the free stream temperature and μ_e is the effective viscosity.

The equation of continuity is satisfied if we choose a stream function $\Psi(x, y)$ such that

$$u=\frac{\partial\Psi}{\partial y}\quad\text{and}\quad v=-\frac{\partial\Psi}{\partial x}$$

Introducing the usual similarity transformation

$$\eta=y\sqrt{\frac{m+1}{2}}\sqrt{\frac{u_0x^{M-1}}{\nu}},\qquad \zeta=\frac{\nu_c x^{1-M}}{Ku_0},\quad \nu_c=\mu_c/\rho$$

$$\Psi(x,y)=\sqrt{\frac{2}{M+1}}\sqrt{\nu u_0x^{M+1}}f(\zeta,\eta)$$

$$\theta(\zeta,\eta)=\frac{T-T_\infty}{\frac{xq_w(x)}{k}\sqrt{\frac{2}{m+1}}\sqrt{\frac{\nu}{u_0x^{M+1}}}}$$

Eqs. (2) and (3) can be written as

$$\frac{1}{\varepsilon}f'''+\frac{2M}{(M+1)\varepsilon^2}f'^2+\frac{1}{\varepsilon^2}ff''-\frac{2\zeta}{M+1}\left[f'-\gamma f'^2\right]=\frac{2(1-M)\zeta}{(1+M)\varepsilon^2}\left(f'\frac{\partial f'}{\partial\zeta}-f''\frac{\partial f}{\partial\zeta}\right) \tag{5}$$

$$\theta''+Pr\left[f\theta'-\frac{2\lambda+1-M}{1+M}f'\theta\right]=\frac{2(1-M)\zeta}{1+M}\left(\frac{f'\partial\theta}{\partial\zeta}-\theta'\frac{\partial f}{\partial\zeta}\right) \tag{6}$$

The boundary conditions are

$$\eta=0,\quad f'(\zeta,0)=1,\quad f(\zeta,0)=0,\quad \theta'(\zeta,0)=-1$$

$$\eta\to\infty,\quad f'(\zeta,0)=0,\quad \theta(\zeta,0)=0 \tag{7}$$

Table 2
$Nu/\sqrt{Re}$ for different values of M at $\lambda = 1$, $\zeta = 1$, $\gamma = 2.01$ and $Pr = 3$

$Nu/\sqrt{Re}$	2.1461	2.0797	2.0261	1.9448	1.8864	1.7596	1.7013
M	−0.2	0.0	0.2	0.6	1.0	3.0	3.75

where the primes denote partial differentiation with respect to η, $Pr = \nu/\alpha_e$ is the Prandtl number, $\gamma = \frac{cu_0 K x^M}{\nu_e}$, is the dimensionless inertia parameter expressing the relative importance of the inertia effect.

3. Numerical method

In this study, the Keller's box finite-difference method was used. Eqs. (5) and (6) associated with

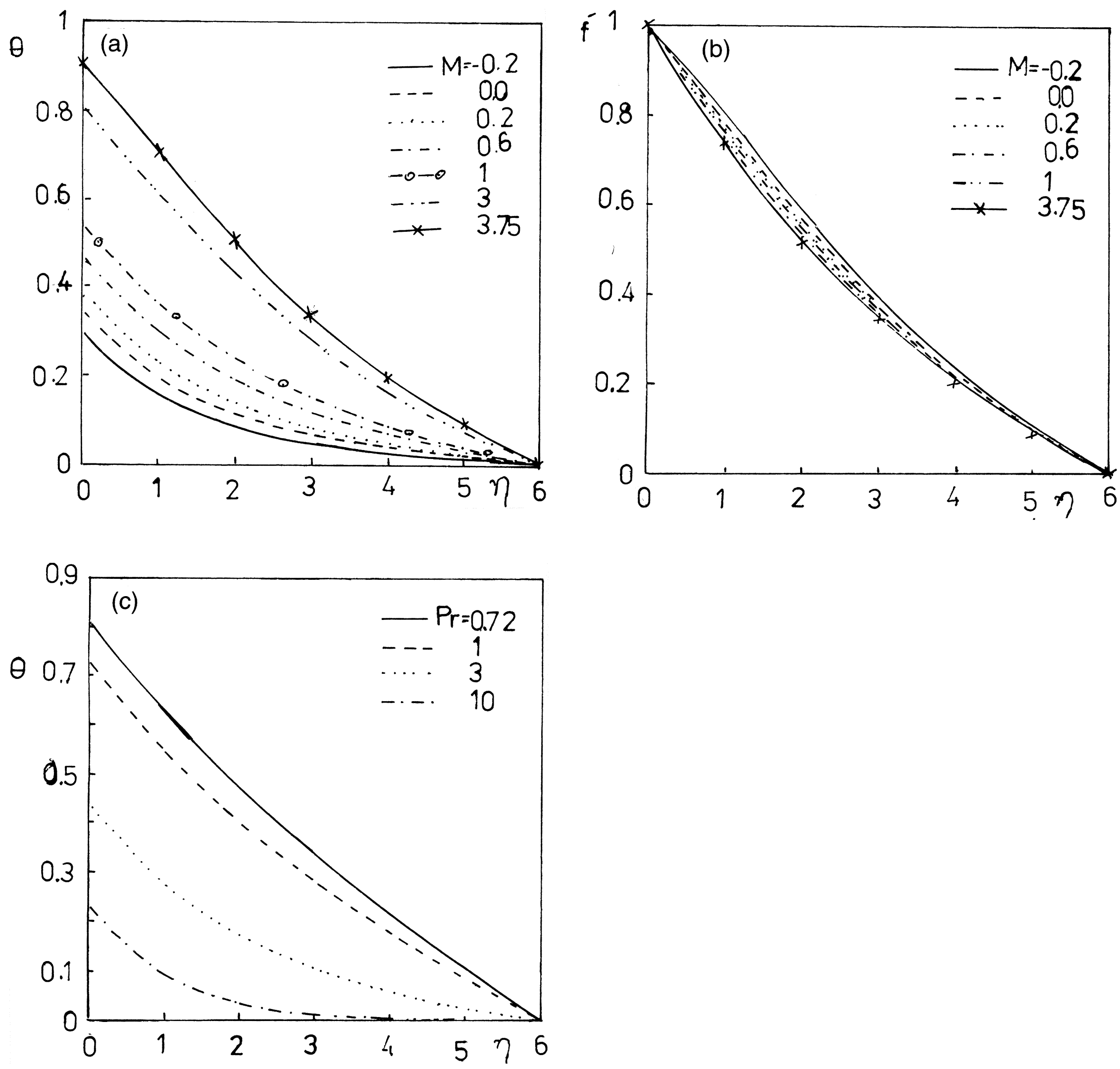

Fig. 3. Temperature profiles (a) and velocity profiles (b) as a function of η for various values of M at $\varepsilon = 0.45$, $\gamma = 2.01$, $\lambda = 1, \zeta = 1$ and $Pr = 3$. (c) Temperature profiles as a function of η for different values of Pr at $M = 0.5$, $\varepsilon = 0.45$, $\lambda = 1$, $\gamma = 2.01$ and $\zeta = 5$.

Table 3
$Nu/\sqrt{Re}$ for different values of Pr at $M = 0.5$, $\varepsilon = 0.45$, $\zeta = 5$, $\lambda = 1$, and $\gamma = 2.01$

$Nu/\sqrt{Re}$	1.0722	1.1987	1.9816	3.825
Pr	0.72	1	3	10

Table 4
$Nu/\sqrt{Re}$ for different values of γ at $M = 1$, $\lambda = 1$, $\varepsilon = 0.45$, $\zeta = 5$, and $Pr = 3$

$Nu/\sqrt{Re}$	1.8116	1.8281	1.9001
γ	0.35	0.73	2.01

boundary conditions (7) were solved by an efficient and accurate implicit finite-difference method similar to that described in Cebeci and Bradshaw [8]. This numerical scheme has several very desirable features that make it appropriate for the solution of parabolic partial differential equations. These features include a second-order accuracy with arbitrary ζ and η spacings, allowing very rapid ζ variations and easy programming of the solution of a large number of coupled equations. For the sake of brevity, details of the solution procedure by this method are not repeated here.

4. Results and discussion

The local Nusselt number $Nu = \frac{q_w(x)(T_w - T_\infty)}{k}$ is given by

$$\frac{Nu}{\sqrt{Re}} = \frac{\sqrt{(M+1)/2}}{\theta(0)}, \qquad Re = \frac{u_0}{\nu} x^{M+1}$$

Numerical calculations are carried out for fluid having Prandtl number equal to 3 with various values of λ and M, γ, ζ at $\varepsilon = 0.45$.

Fig. 2 shows the effect of changing parameter λ for a selected value of $M = 0.5$, $Pr = 3$, $\zeta = 3$ and $\gamma = 2.01$. It is clear that the temperature decreases with increasing heat flux exponent and the heat is transferred from the continuous stretching surface to the fluid medium. From Table 1, the dimensionless heat transfer coefficient $Nu/\sqrt{Re}$ increases with increasing heat flux exponent λ. Thus, a higher value of λ indicates a higher heat transfer rate from the surface.

It is seen from Fig. 3(a), the effect of increasing M on the temperature profiles for $\gamma = 2.01$, $\gamma = \zeta = 1$ and $Pr = 3$. It is clear that the temperature decreases with increasing exponent of velocity M and for these values of M heat is transferred from continuous surface to the fluid medium. Furthermore, the thermal boundary layer is also increasing with increasing M and then more heat is dissipated to the fluid medium.

The dimensionless velocity depends on the parameter M where $u = \frac{\partial \Psi}{\partial y} = U_0 x^M f(\eta)$. Fig. 3(b) shows samples of the dimensionless velocity for $\gamma = 2.01$, $\lambda = \zeta = 1$ and $Pr = 3$ as a function of η for various values of exponent of velocity M. These curves show that the velocity profiles decrease with increasing the exponent of velocity M from -0.2 to 3.75. In other words, the momentum boundary layer thickness increases as M decreases. However, there is no solution for the plate moves away from the origin for $M > 3.75$ and $M < -0.2$, where the flow part of the problem exhibit

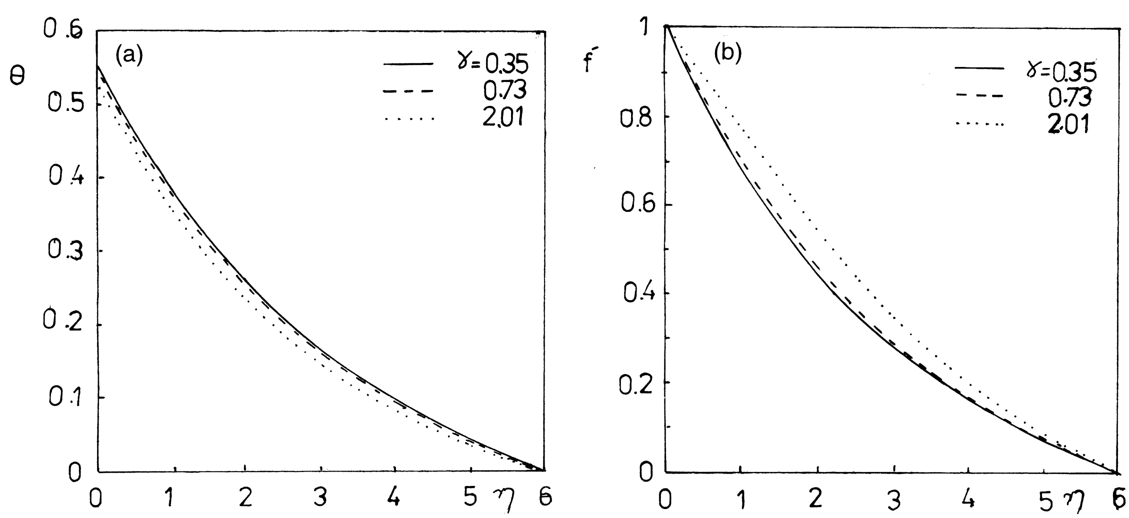

Fig. 4. Temperature profiles (a) and velocity profiles (b) as a function of η for different values of γ at $M = 1$, $\varepsilon = 0.45$, $\lambda = 1$, $\zeta = 5$ and $Pr = 3$.

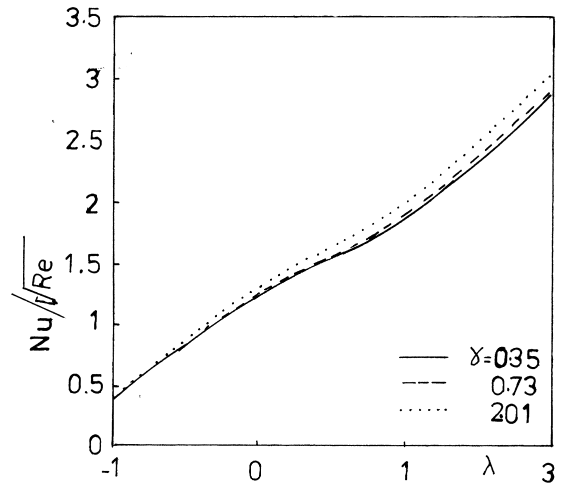

Fig. 5. Variation of $Nu/\sqrt{Re}$ as a function of λ for different values of γ at $M = 0.5$, $\varepsilon = 0.45$, $\zeta = 5$ and $Pr = 3$.

no similarity solutions for this range of M as explained by Banks [9].

From Table 2, the dimensionless heat transfer coefficient $Nu/\sqrt{Re}$ decreases with increasing exponent of velocity M.

Fig. 3(c) shows samples of the dimensionless temperature as a function of η for various values of Prandtl number. The temperature decreases with increasing Prandtl number.

From Table 3, the dimensionless heat transfer coefficient $Nu/\sqrt{Re}$ increases with increasing Prandtl number. This table indicates that increasing Prandtl number enhances the heat transfer coefficient, where thermal boundary layer decreases.

Some samples of the resulting velocity and temperature profiles for $M = 1$, $\lambda = 1$, $\zeta = 5$ and $Pr = 3$ for different values of γ in Fig. 4(a) and (b). It can be seen from Fig. 4(a) and (b) that the temperature decreases and velocity increases with increasing γ. We can conclude that boundary and inertia effects tend to increase the velocity and decrease the wall temperatures.

From Table 4, the dimensionless heat transfer coefficient $Nu/\sqrt{Re}$ increases with increasing γ.

The heat transfer coefficient in the dimensionless form of $Nu/\sqrt{Re}$ is presented in Fig. 5 as a function of λ in the range $-1 \leq \lambda \leq 3$ for different values of γ. These curves show that the parameter λ increases the heat transfer rate greatly, while the parameter γ increases it slightly. The parameters γ and λ enhance the heat transfer coefficients.

In order to verify the numerical accuracy of the solution, numerical results were first obtained for the case $\varepsilon = 1$ and $\zeta = 0$, and compared to those reported by Ali [5], as shown in Figs. 6–8. The fact that these results show a close agreement is an encouragement for further study of the effects of other various parameters on the moving plate.

From Figs. 6 and 7, it is clear that the numerical

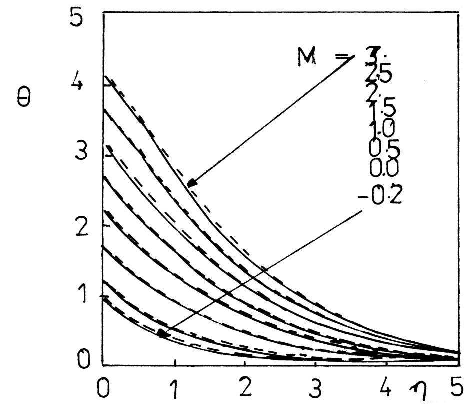

Fig. 6. Temperature profiles as a function of η at different values of M. - - - results of Ali [5]; ——— present study.

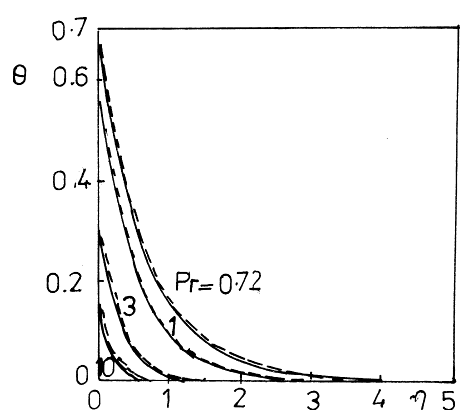

Fig. 7. Temperature profiles as a function of η at different values of P_r. - - - results of Ali [5]; ——— present study.

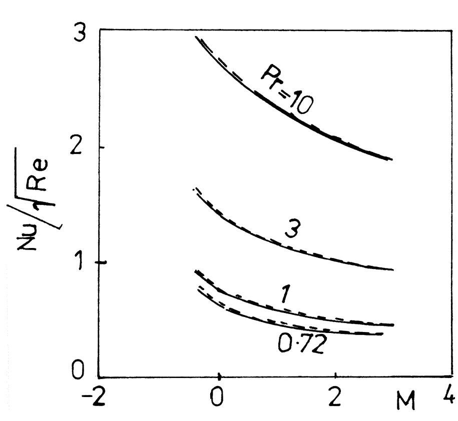

Fig. 8. Variation of $Nu/\sqrt{Re}$ as a function of M at different values of P_r. - - - results of Ali [5]; ——— present study.

solution gives good results in comparison with numerical solution by Ali [5].

5. Conclusions

The heat transfer over a continuously moving plate embedded in non-Darcian porous medium with variable surface heat flux and velocity u_w has been solved for $-0.2 \leq M \leq 3.75$ and $-1 \leq \lambda \leq 3$. It was found that the parameters λ and γ enhance the heat transfer coefficient. The dimensionless heat transfer coefficient decreases with increasing exponent of velocity M, whereas the dimensionless heat transfer coefficient increases with increasing exponent of heat flux, Prandtl number and inertia parameter. The dimensionless temperature decreases with increasing exponent of heat flux, Prandtl number and inertia parameter, whereas the dimensionless temperature increases with the exponent of velocity.

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